OPTIMAL DESIGN OF A MODIFIED ORTHOGLIDE PARALLEL KINEMATIC MECHANISM USED IN A CNC MILLING MACHINE

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ABSTRACT
This paper presents a method for determining the design parameters of a modified orthoglide mechanism integrated into a CNC milling machine through an optimization process. The optimization method is used to find the geometric parameters of the modified orthoglide such that the workspace, manipulability and stiffness are maximized. The optimization process is performed with the aid of a genetic algorithm, considering the geometric limits imposed by the milling machine, such as the actuator lengths limits.

KEYWORDS: Orthoglide, Optimization, Genetic algorithm, Milling machine

1. INTRODUCTION
Parallel kinematic machines (PKM) are commonly used for machining processes because of their advantages such as high stiffness, low inertia, high accuracy and also the ability to have increased number of freedoms compared to classic machine tools. The construction of such a machine tool into a university laboratory for experimental and educational purposes requires high cost. In order to construct and demonstrate a PKM with low cost, the facilities (actuation, control, hardware, software) of a 3D CNC milling machine existing into the laboratory can be used [1]. For that purpose an orthoglide parallel mechanism with three degrees of freedom (dof) is used /2, 3/, where the linear joints are actuated and controlled by the conventional 3D CNC milling machine. The geometry of this mechanism makes it ideal to be placed in the milling machine existing into the laboratory with minimum modifications.

In the present paper a 3-dof modified orthoglide parallel mechanism integrated into a CNC machine is considered. The main problem is to find the optimum design parameters of the mechanism, considering the limits imposed by the CNC milling machine geometry such that the workspace, manipulability and stiffness of the orthoglide are maximized. This problem is investigated in the present paper where for the optimal design of the orthoglide mechanism a genetic algorithm is used.

2. MATHEMATICAL FORMULATION
The orthoglide mechanism considered in the paper is shown in Figure 1. The mechanism consists from the base, a moving platform and three identical fixed-length limbs, connected to the base by prismatic joints. Each of the three limbs contains one parallelogram. The three parallelograms are connected to the moving platform and the sliders by revolute and universal joints respectively. The drive and the control of the prismatic joints in x, y and z directions are performed by CNC milling machine actuators. The moving platform of the mechanism has three translational dof with respect to the base, so that it retains a constant orientation during the motion. A serial 2-dof passive mechanism (3 in Figure 1) is attached for decoupling of the CNC machine x and y axes.
Kinematic analysis of the mechanism is based on its geometric model (Figure 2), where each parallelogram is illustrated as a unique rod. All mechanism parameters are defined as shown in Figure 2 where the $A_B$, $R_i$, and $r_i$ are the scalar variables of the actuated prismatic joints with length $d_i$, $R_i$, and $r_i$ are the distances $OA_i$, $R_i$, and $r_i$ of the base, $r_i$ the distances $O_P$, $r_i$ of the moving platform and $c_i$ are the limbs length $B_iP_i$. $T_B$ and $T_T$ are the tool base and the tool tip respectively.

$$A_{Op} = [B_x \ B_y \ B_z]'$$

$$A_{Pt} = [x_T \ y_T \ z_T]'$$

$$p_{Pt} = [s \ 0 \ h]'$$

Figure 1: Modified orthoglide: 1 - connection parts of the orthoglide to the CNC machine (Base); 2 – moving platform; 3 – arm; 4 – limbs.

Figure 2: Geometric model of the mechanism.
The coordinates frames \( \{A\} \) and \( \{P\} \), attached to the base and mobile platform respectively, are always parallel due to the mechanism’s structure. The leading superscripts indicate the coordinate frame to which the vectors refer to. For the inverse kinematic analysis of the orthoglide mechanism the geometrical model presented in Figure 2 is used. In this case the position vector \( A^P \) of the moving platform is given and the displacement of the prismatic joints \( d_i, (i = 1, 2, 3) \) is determined.

For each kinematic chain the next vector-loop equations are defined.

\[
\begin{align*}
OA_i + A_iB_i + B_iP_i &= OOP_i + OP_iP_i \quad \text{for } i = 1, 3 \\
OA'_i + A'_iA_i + A_iB_i + B_iP_i &= OOP_i + OP_iP_i \quad \text{for } i = 2
\end{align*}
\]  

(1) (2)

Since the length of each leg \( c_i \) \( (i = 1, 2, 3) \) is constant the following constraint equation can be written:

\[
|BP_i| = c_i^2 \cdot c_i = c_i^2, \text{ for } i = 1, 2, 3
\]  

(3)

The following relations are derived:

\[
\begin{align*}
d_1 &= p_x + R_1 - \eta \pm \sqrt{c_1^2 - p_y^2 - p_z^2} \\
d_2 &= -p_y + R_2 - r_2 \pm \sqrt{c_2^2 - (p_x - d_1)^2 - p_z^2} \\
d_3 &= -p_z + R_3 - r_3 \pm \sqrt{c_3^2 - p_x^2 - p_y^2}
\end{align*}
\]  

(4)

The relation between the input (actuated joints) velocities \( \dot{q} = [\dot{d}_1, \dot{d}_2, \dot{d}_3]^{T} \) and the moving platform output velocities \( \dot{x} = [p_x, p_y, p_z]^{T} \) yields:

\[
J_x \dot{x} = J_x \dot{q}
\]  

(5)

The Jacobian matrix \( J \) is defined as \( J = J_x^{-1} J_x \) and is given by the equation:

\[
J = \begin{bmatrix}
1 - \frac{(p_x - d_1)^2}{a_{11} \cdot a_{22}} & \frac{p_y + (p_x - d_1)}{a_{11}} & \frac{p_z + (p_x - d_1)}{a_{11} \cdot a_{22}} \\
\frac{(p_x - d_1)}{a_{22}} & -1 & \frac{p_z}{a_{22}} \\
\frac{p_x}{a_{33}} & \frac{p_y}{a_{33}} & -1
\end{bmatrix}
\]  

(6)

where \( a_{11} \), \( a_{22} \) and \( a_{33} \) are:

\[
\begin{align*}
a_{11} &= p_x - d_1 + R_1 - \eta \\
a_{22} &= -p_y - d_2 + R_2 - r_2 \\
a_{33} &= -p_z - d_3 + R_3 - r_3
\end{align*}
\]  

(7)

The orthoglide mechanism is free of inverse and direct kinematics singularities inside the workspace, because both matrix determinants \( J_x \) and \( J_q \) are not equal to zero inside the workspace. This fact makes the orthoglide an ideal mechanism for use in machining processes.

The design variables of the orthoglide mechanism are the base length \( R_i \), the moving platform length \( r_i \) and the limbs length \( c_i \). The variables are optimized within specific limits imposed by the
CNC milling machine geometry taking into account the limits of the actuated prismatic joints. The design variables of the orthoglide mechanism integrated into the CNC milling machine have to assure the basic performances of workspace, manipulability and stiffness.

The aim of the present paper is to determine the design variables of the mechanism in order to maximize the workspace volume, the conditioning index and the stiffness of the mechanism.

Thus an optimization problem can be formulated with the following objective function:

\[ F = a^* F_1 + b^* F_2 + c^* F_3 \]  (8)

where \( F_1 = \left( \frac{k_1}{k_2} \right) - k_3, F_2 = n_1 \cdot CI_{RMS}, F_3 = m_1 - m_2 \cdot \log_\left( \| D_{\text{max}} \| \right)_{RMS} \) with \( a, b \) and \( c \) are the weighting factors and \( k_1, k_2, k_3, n_1, m_1 \) and \( m_2 \) are the factors for the normalization of the workspace \( V \), conditioning index number \( CI_{RMS} \) and maximum deformation \( \| D_{\text{max}} \| \) respectively.

To determine the workspace of the mechanisms a variation of the box method/5, 6/ is used. The box size affects the accuracy of the formed workspace and as well the computational time. The choice of the box size is made by use of a trial and error method, in order to achieve good workspace accuracy with fast computational time. In the present paper a sub box of size 4 mm was used.

The manipulability of the mechanism is estimated through the conditioning index number \( CI \), which is the reciprocal of the condition number of the Jacobian matrix \( J \)/6, 7/. The purpose of the optimization will be the \( CI \) to reach value 1, if this is possible. In the present paper the RMS value of all the \( CI \) is used that is given by:

\[ CI_{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} CI_i^2} \]  (9)

The stiffness matrix \( K \) relates the external force vector \( \tau \) at the moving platform to the output displacement vector \( D \) of the moving platform according to equation/8/:

\[ \tau = K \cdot D \]  (9)

The deformation \( \| D_{\text{max}} \| \) is used to evaluate the stiffness of the manipulator, which is defined as the local stiffness index (LSI). Smaller maximum deformation means higher stiffness. The value of the \( \| D_{\text{max}} \| \) depends on the position of the mechanism. For that reason in the present paper the RMS value of all the \( \| D_{\text{max}} \| \) is used that is given by:

\[ \| D_{\text{max}} \|_{RMS} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \| D_{\text{max},i} \|^2} \]  (10)

3. OPTIMIZATION METHOD

The optimization problem of the orthoglide design variables is solved with a variation of the simple genetic algorithm (SGA)/4/. Basic differences are the multi point crossover, adaptive mutation rates, hill climbing method (HCM) and constraints handling method (CHM) for the variables that are optimized. These variables are the base length \( OA_i \), the moving platform length \( O_P \), and the limbs length \( B_P \) \( (i=1, 2, 3) \) of the orthoglide mechanism. The input data for the algorithm are the variables bounds, the actuated joints limits and the genetic algorithm parameters. In these parameters are included the initial SGA parameters such as population size, cross over rate, initial mutation rate, number of generations. The limits for the base, the moving platform
and the limbs length are imposed from the milling machine limits. Also the moving platform can not be larger than the base length in order to assure the stability of the mechanism. The fitness function is defined by the equation (6), which is used in all steps of the algorithm. The three criteria of the fitness function workspace, manipulability and stiffness are all normalized. For the normalization of the values the trial and error method is used to limit the values to fluctuate into the same bounds.

The differences between the basic genetic algorithm and the one used in the present paper are the following. First a CHM method is used to eliminate the variables that are outside the constraints limits. There is also a handling method for extreme values of the optimized parameters LCI and LSI. If the value is outside of the predefined limit then a new value is set which is the upper or the lower limit. Then the optimum individual is selected and is processed with HCM a local search method to achieve better results. Finally the results are examined and if they are satisfactory the optimization procedure is finished. In other case the optimization procedure begin again with different parameters like crossover rate, number of generation or different random first generation.

4. RESULTS

The optimization process takes into account the limits of the three design variables which are base length, moving platform length, limbs length and the actuated joint limits imposed by the CNC milling machine. The limits and the initial genetic parameters are presented in Table 1.

<table>
<thead>
<tr>
<th>Design variables bounds</th>
<th>Actuated joints limits</th>
<th>Initial genetic algorithm parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base length 260 mm ≤ R ≤ 300 mm</td>
<td>0 mm ≤ d; ≤ 150 mm i = 1, 2, 3</td>
<td>Crossover rate = 0.8</td>
</tr>
<tr>
<td>Moving platform length 0 mm ≤ r; ≤ 40 mm</td>
<td></td>
<td>Probability of mutation = 0.05</td>
</tr>
<tr>
<td>Limbs length 180 mm ≤ c; ≤ 220 mm</td>
<td></td>
<td>Population size = 60 atoms</td>
</tr>
<tr>
<td>and (R - r) &gt; 100 mm</td>
<td></td>
<td>Number of generations = 500</td>
</tr>
</tbody>
</table>

As shown in Table 1 the base length must always be 100 mm longer than the moving platform in order to assure the stability of the mechanism.

The proposed algorithm is applied for the optimum design of an orthoglide mechanism to be integrated into a CNC milling machine considering three optimization criteria. Four cases are presented. The first case takes into account the optimization of the mechanism to maximize only the workspace. The weighting factors are respectively a=1, b=0 and c=0. The second case takes into account the optimization of the conditioning index number only, with a=0, b=1 and c=0. The third case the optimization of stiffness is taken into account with a=0, b=0 and c=1. Finally the optimization considering all three criteria is performed with a=1, b=1 and c=1. In all cases the factors used in equation (8) are \( n_l = 100, k_1 = 7.15, k_2 = 10^5, k_3 = 28.6 \) and \( m_1 = 80 \) and \( m_2 = 25 \). These values are defined by the trial and error method. The optimization results are presented in Table 2.

In the present paper the accuracy for computing and scanning the workspace was 4 mm as described in /6/ in order to reduce the computational time without deteriorating the workspace accuracy. To avoid the singularity poses that appear near the boundaries of the workspace approximately 95 percent of the workspace volume is used.
Table 2: Optimization results.

<table>
<thead>
<tr>
<th>No.</th>
<th>Weighting factors</th>
<th>Base length (mm)</th>
<th>Moving platform length (mm)</th>
<th>Link length (mm)</th>
<th>Condition index</th>
<th>Max RMS deformation</th>
<th>Workspace (mm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(a = 1) (b = 0) (c = 0)</td>
<td>297.75</td>
<td>3.9375</td>
<td>215.625</td>
<td>0.55</td>
<td>6.17E+17</td>
<td>2774208</td>
</tr>
<tr>
<td>2</td>
<td>(a = 0) (b = 1) (c = 0)</td>
<td>300</td>
<td>27.875</td>
<td>220</td>
<td>0.65</td>
<td>3.41</td>
<td>2343808</td>
</tr>
<tr>
<td>3</td>
<td>(a = 0) (b = 0) (c = 1)</td>
<td>294.688</td>
<td>25.3125</td>
<td>220</td>
<td>0.648</td>
<td>3.25</td>
<td>2318720</td>
</tr>
<tr>
<td>4</td>
<td>(a = 1) (b = 1) (c = 1)</td>
<td>297.813</td>
<td>28.125</td>
<td>219.375</td>
<td>0.648</td>
<td>3.28</td>
<td>2317568</td>
</tr>
</tbody>
</table>

In all numerical examples the CI number takes values above 0.5 which assures that the mechanism will be fully controllable and agile. Also the maximum value of the deformation takes very low values except when only the workspace criterion is applied where the deformation takes an extremely high value. In all cases, as shown in Table 2, the base and the link length are near their upper limits. The moving platform length has the same behavior except when the workspace is to be maximized where the value reaches the lower limit.

The distribution of the conditioning index for a specific z-axis level is presented in Figure 3, when all the criteria are used. As it is expected in case of the optimized lengths the CI number is maximized towards the center of the workspace and the mechanism becomes more controllable. In Figure 4 the deformation of the optimized modified orthoglide is presented for a given z-axis level of the workspace, when all the criteria are used. It is noticed that the deformation of the mechanism is minimized near the center of the workspace and takes higher values towards the outer limits of the workspace. This expected distribution means that the stiffness of the machine is increased near the center of the workspace.

Figure 3: Conditioning index number of the 4th case mechanism for z-axis level 10 mm.

Figure 4: Deformation of the 4th case mechanism for z-axis level 10 mm.

Finally two pictures of the parallel mechanism fitted into the 3D CNC milling machine are presented in Figure 5.
5. CONCLUSIONS

The integration of an orthoglide type mechanism into a 3D CNC milling machine (Lagun 8000M) is presented in the present paper. The integration is performed through an optimization method which uses genetic algorithms. The optimization results show that the genetic algorithm can be guided to maximize each one of the three criteria and all at the same time with little or no effort, just by changing the weighting factors. This fact makes the optimization tool flexible, agile and minimum time consuming.

It also noted that the differences between the results are in all cases indistinguishable. This happens because the limits of the optimization are very confined. This is the reason that almost all the variables reached their upper limits. The only variable that did not reached its upper limit was the moving platform. It is also noted that the moving platform plays a significant role in the maximum deformation. When it took a value near the lower limit then the deformation was unacceptably high. The above behavior makes the optimization tool suitable for mechanisms with higher length limits and more complex kinematics.

6. REFERENCES

8. Xin-Jun Liu, Optimal kinematic design of a three translational DoFs parallel manipulator, Robotica, 00 (2005) 1-12.